ARKIV FÖR FYSIK Band 11 nr 10

Communicated 14 March 1956 by Erik Hulthén and Ivar Waller

On the identification of stopping particles in nuclear emulsions

By Sigward Nilsson

With 12 figures in the text

Summary

Measurements of constant sagitta, mean gap length versus residual range and photometric density versus residual range, have been carried out on 24 stopping protons and 10 π-mesons in G5 emulsion. The Fay-Gottstein-Hain $p = 0.5\ \mu$ (protons) and $\pi = 0.5\ \mu$ (π-mesons) scattering scheme is calibrated and the conversion factors deduced when calculating the second differences from the third and fourth differences respectively. The photometric density is normalised to minimum tracks and the absolute normalised photometric coefficient and gap coefficient are compared. A linear approximation to the points of photometric density versus residual range in a log-log plot is shown to give reasonable values for sufficiently large ranges. An apparatus for measuring photometrically ionisation of tracks in nuclear emulsions adapted for the Cooke Microscope, is briefly described.

1. Introduction

The discovery of a variety of new particles and modes of decay together with the use of the new technique of stripped emulsions has made it important to identify stopping particles in the emulsions. The possibility of stopping particles has increased, and one can follow particles from different events through the stack. The energy of the stopping particle can be determined very accurately by measuring the range only, provided its mass and charge is known. The charge of the particle can be determined by δ-ray counting etc. We restrict ourselves to singly charged particles, however. The mass of the particle is another quantity that can be measured. The methods most used at present to determine the mass of the stopping particle are based on measuring multiple Coulomb scattering, measuring mean gap length versus residual range and photometric measurement of ionisation in the track as a function of the residual range. Electron sensitive emulsions which begin to saturate at $\beta = 0.4$ for protons, are mostly employed by workers in the field, and thus older methods such as blob counting–residual range cannot be used at the end of the track. The method of gap counting–residual range can be used but is unreliable.

The scattering method has the advantage of being independent of any measurements of ionisation, on the other hand it is usually connected with large statistical errors and is sensitive to distortion. All the other methods are based on some estimate of the energy losses of the particle in traversing the emulsion.

The present investigation has been undertaken to calibrate the Fay-Gottstein-Hain scattering schemes and to compare the photometric method and that of mean gap length as a measure of ionisation in the determination of the particle mass. Fowler and Perkins [17] have used the method of blob and gap counting with great success on fast tracks (proton ranges greater than 1 cm) but this method is not applicable to the ends of the tracks where the blob density saturates. It was thought that a new investigation of the mean gap length near the end of the tracks would be complementary to that of Fowler and Perkins. In such an experiment it is important to know the exact mass of the particle independently of the measurements otherwise extreme signals may be unfairly excluded. The investigation is based on measurements on 24 protons and 10 π-mesons in an Ilford G5 emulsion, 600 μ thick, minimum blob density 23.9 blobs/100 μ (Sardinia flight no. 21, 1953). The π-mesons are uniquely identified from complete π-μ-e decays. The protons are identified by the consistency of the three mass determinations. Two of these are strictly independent. In view of the experimental standard deviations one would expect only ~ 1 particle out of 24 to give a mass value of that of a deuteron at one particular range. Measurements are, however, made at 13 different ranges in, for example, the determination of the mean gap length and the likelihood of this one track giving an extreme signal at all these different points is negligible. The proton sample therefore is a pure one.

The distribution of track lengths of the observed particles can be seen in Fig. 1. All measurements have been performed with a Cooke, Troughton & Simms M 40180 Nuclear Microscope.

2. Constant Sagitta method

(a) Theory

From the scattering theory it is well known that

$$\frac{\bar{D}_2}{t}\,\frac{180}{\pi}=K\,\frac{z}{p\beta c}\left(\frac{t}{100}\right)^{\frac{1}{2}} \tag{1}$$

where t is the cell-size in μ, K the scattering constant, ze the charge, p the momentum and βc the velocity of the particle. $\bar{D}_2$ is the absolute mean value of the second differences $D_i^{(2)}$ in μ. No smoothing factor has been taken into account. Fowler's coordinate method is used. The second and higher order differences can be calculated from

$$\begin{aligned} D_i^{(2)} &= y_{i+2}-2y_{i+1}+y_i \\ D_i^{(3)} &= D_{i+1}^{(2)}-D_i^{(2)} \\ \bar{D}_n &= \langle\,|D_i^{(n)}|\,\rangle \end{aligned} \tag{2}$$

y_i are the successive readings at points p_i of the track at intervals t along the x-axis.

Scott [1] has given the distribution function of $D_i^{(2)}$, and one can show [2] that the mean absolute values of the second, third and fourth differences are related in the following manner,

$$\begin{aligned} \langle\,|D_i^{(2)}|\,\rangle &= \bar{D}_2 \\ \langle\,|D_i^{(3)}|\,\rangle &= \bar{D}_3=\sqrt{\frac{3}{2}}\,\bar{D}_2 \\ \langle\,|D_i^{(4)}|\,\rangle &= 2\,\bar{D}_2. \end{aligned} \tag{3}$$

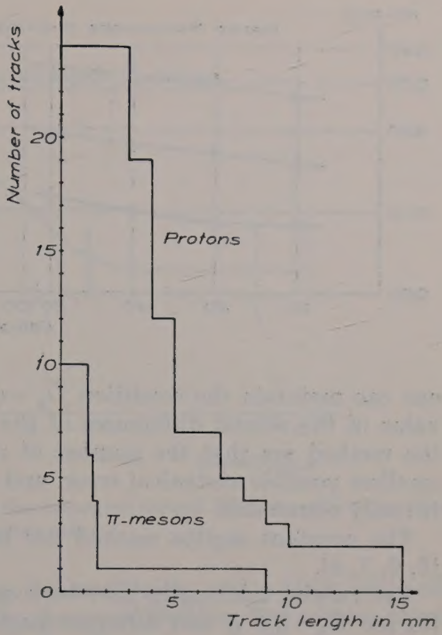


Fig. 1. Distribution of track lengths of the measured particles.

$\bar{D}_2$ measures the deviation of the track from a straight line, $\bar{D}_3$ the deviation from a circle with an arbitrary radius and $\bar{D}_4$ the deviation from a curve with linearly varying curvature. Thus it is possible to eliminate C-shaped and S-shaped distortion in the emulsion by calculating the third and fourth differences respectively.

The measured mean value $\bar{D}_n$ consists of true scattering $\bar{D}_n^*$ and noise δ_n. If the distributions are normal and true scattering and noise independent, the noise can be eliminated according to

$$\bar{D}_n = \sqrt{(\bar{D}_n^*)^2 - (\delta_n)^2}. \tag{4}$$

This is approximately true even for higher order differences (S. Biswas *et al.* [2]).

The following relation holds between the kinetic energy of a particle (charge ze, mass M) and its range R,

$$T = a z^{2n} M^{1-n} R^n. \tag{5}$$

For G5 emulsion with T in MeV, M in proton masses, R in μ the constants are $a = 0.281$ $n = 0.568$ (3, 4).

Equations (1) and (5) together with the relativistic expression,

$$\frac{p\beta c}{T} = \frac{\beta^2}{1-\sqrt{1-\beta^2}} = \gamma(\beta) \tag{6}$$

yield

$$\bar{D}_2 = \frac{\pi}{1800} \frac{K}{a z^{2n-1} M^{1-n} R^n \cdot \gamma} t^{3/2}. \tag{7}$$

By choosing,

$$t^{3/2} \propto (1/K)\, \gamma z^{2n-1} M^{1-n} R^n \tag{8}$$

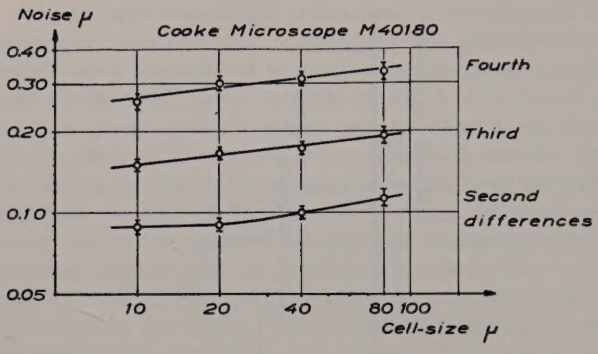


Fig. 2. The variation of noise with cell-size for small cells.

one can maintain the condition $\bar{D}_2 = \text{const.}$ along the track, i.e. the absolute mean value of the second differences of the sagitta is kept constant. The advantages of the method are that the number of cells is the largest possible, which means the smallest possible statistical error, and a constant signal to noise ratio, so the noise is easily eliminated.

The constant sagitta method has been developed in a number of recent papers [5, 6, 7, 8].

The varying cell lengths have been calculated by H. Fay, K. Gottstein and K. Hain [9] for $\bar{D}_2 = 0.5\ \mu$ and different particles, protons, τ- and π-mesons ($M_p = 1836$, $M_\tau = 965$, $M_\pi = 273\ m_e$), taking into account the variation of the scattering constant with cell-size and energy and a $4\bar{D}$ – cut-off and assuming a constant noise-level. $\bar{D}_2 = 0.5\ \mu$ has been chosen because it gives a signal to noise ratio about 5. Let (M_0, $\bar{D}_{20} = 0.5\ \mu$) be the mass and the absolute mean value of the second differences respectively, for the reference particle, and (M, $\bar{D}_2$) the corresponding values for an unknown particle. From equation (7) it can be seen that in the first approximation the following relation holds,

$$\frac{M}{M_0} = \left(\frac{0.5}{\bar{D}_2}\right)^{\frac{1}{1-n}}. \tag{9}$$

One obtains the relative error in the mass determination,

$$\frac{\sigma(M)}{M} = \frac{1}{1-n} \cdot \frac{\sigma(\bar{D}_2)}{\bar{D}_2} = 2.31\,\frac{\sigma(\bar{D}_2)}{\bar{D}_2}. \tag{10}$$

The relative standard deviation in $\bar{D}_2$ is taken to be $0.75/\sqrt{N}$, where N is the number of second differences. This includes statistical errors only. Because of the amplication factor 2.31 the statistical standard error in the mass is very large, theoretically of the order of 20 % for a proton track of range 5000 μ.

(b) Experimental results

The scattering measurements have been carried out with an eyepiece filar micrometer [9]. It is necessary to know the noise level of the microscope for the small cells employed, down to 10 μ. This has been investigated by measurements on flat

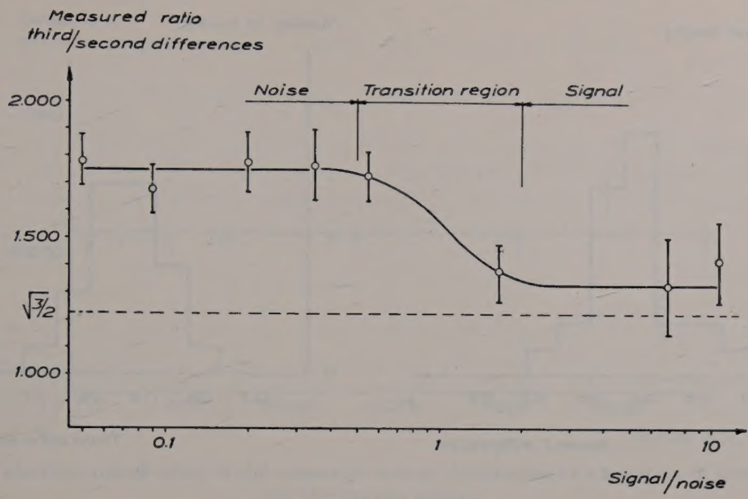


Fig. 3. The ratio of the measured third differences to the measured second differences as a function of the signal to noise ratio.

tracks. The tracks have been measured using the small cells, and a large cell, and the signal has been eliminated according to the theory of Molière. The second, third and fourth differences have been calculated. The result is shown in Fig. 2.

The noise indicated includes reading-, stage-, and grain-noise. The errors are standard errors derived from the experimental distributions. For the first three points 300 second differences have been counted, for the last, 100.

Thus the noise increases with increasing cell size even for small cells. A least square fit to the last three points of the second differences yields $\delta_2 \propto t^{0.16}$ in agreement with published measurements on the Cooke Microscope [10].

Concerning the elimination of noise the following constant values have been adopted:

$$\delta_2 = 0.100\ \mu, \quad \delta_3 = 0.178\ \mu, \quad \delta_4 = 0.315\ \mu \qquad (11)$$

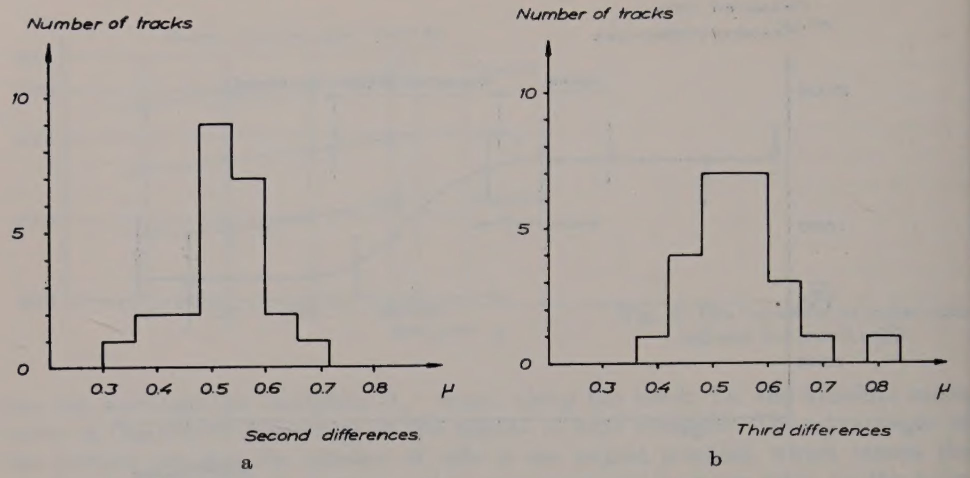
corresponding to a cell $t = 40\ \mu$.

The properties of the noise distributions do not seem to have been investigated previously. The difference between the properties of the noise distributions and the signal distributions is illustrated in Fig. 3.

When obtaining this figure, 5 flat tracks in different emulsions were measured with different cell sizes yielding a total of 2100 second differences. The signal to noise ratio was deduced assuming a $t^{\frac{3}{2}}$ variation of the signal.

When calibrating the Fay-Gottstein-Hain scheme the stopping proton and π-meson tracks were measured with slight smoothing, 2–3 grains, for small cells ($< 40\ \mu$) and ordinary smoothing, 5 grains, for larger cells. Approximately the same number of readings were taken in every track, about 75 for protons and 55 for π-mesons. All mean values were calculated with $4\,\bar{D}$ – cut-off without replacement. Cut-off was applied to each of the second, third and fourth differences separately.

The distributions of the absolute mean values of the differences for the proton tracks are shown in Fig. 4.



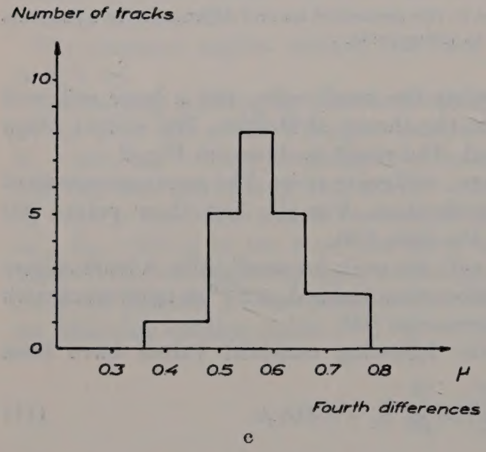


Fig. 4. The distributions of the absolute mean values of the differences among the proton tracks.

(a) The second differences,
(b) $\sqrt{\frac{2}{3}}$ the third differences,
(c) $\frac{1}{2}$ the fourth differences.

The mean values and relative standard deviations are,

$$\langle \bar{D}_2 \rangle = 0.527\ \mu \quad \sigma = 3.1\,\%$$

$$\langle \sqrt{\frac{2}{3}}\,\bar{D}_3 \rangle = 0.545\ \mu \quad \sigma = 3.3\,\%$$

$$\langle \tfrac{1}{2}\bar{D}_4 \rangle = 0.577\ \mu \quad \sigma = 3.1\,\%.$$

The mean value of $\bar{D}_2$ is in fair agreement with calibration experiments already reported [11]. The difference from Friedlander's, Johnston's and Merlin's values, 0.543, 0.539, 0.549 μ, is due to the different cut-off procedure used.

The conversion factors between the $\bar{D}_2$'s calculated, respectively, from the second, the third and fourth differences are,

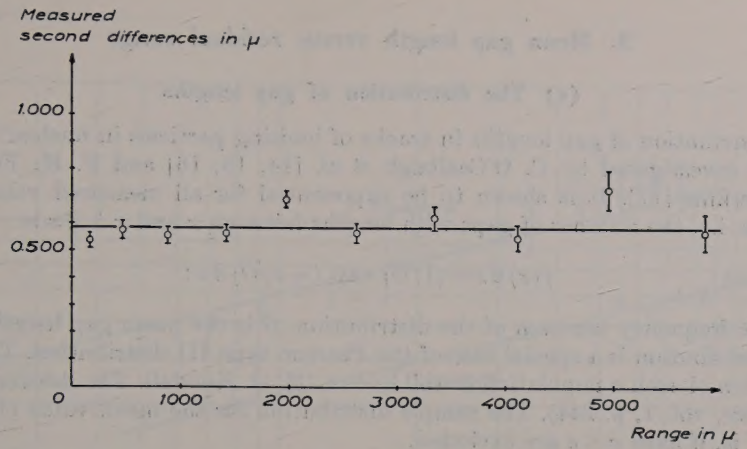


Fig. 5. The absolute mean value of the measured second differences as a function of residual range for the proton group.

$$\frac{\bar{D}_2\ (\text{3rd diff.})}{\bar{D}_2\ (\text{2nd diff.})} = 1.034 \pm 0.027$$

$$\frac{\bar{D}_2\ (\text{4th diff.})}{\bar{D}_2\ (\text{2nd diff.})} = 1.095 \pm 0.031.$$

These values have been calculated without correction for distortion, because it has been shown that distortion is negligible in the emulsions.

In the errors indicated the correlation between the second and following differences in the same track has been taken into account. The correlation coefficients are $\varrho_{23} = 0.66$, $\varrho_{24} = 0.57$ respectively.

These results compare favourably with a calibration experiment using other cell schemes [12]. There is a definite excess when calculating the signal from the third and fourth differences instead of the second and it is necessary to apply the empirical conversion factors.

The corresponding figures for the π-meson tracks are

$$\bar{D}_2 = 0.543\ \mu,\ \left\langle \sqrt{\frac{2}{3}} \cdot D_3 \right\rangle = 0.530\ \mu, \quad \text{and}\ \left\langle \tfrac{1}{2}\ \bar{D}_4 \right\rangle = 0.546\ \mu.$$

Since they are not of sufficient statistical significance they have not been used when calculating the conversion factors. They show, however, that it is necessary to calibrate the $\pi = 0.5\ \mu$ scheme too, and that the deviation from the expected $0.5\ \mu$ is of the same order as for the proton group.

The condition $\bar{D}_2 = \text{const.}$ along the track must be fulfilled if the scattering scheme is correct. The variation of the absolute mean value of the measured second differences along the track is shown in Fig. 5 for the proton group.

A weighted least square fit to the points gives $\bar{D}_2 \propto R^{0.000012}$ thus confirming, within the limits of experimental error, that $\bar{D}_2 = \text{const.}$ for the P-scheme. This line is shown in the diagram. This point has also been carefully investigated by S. Kaneka using μ-mesons [13]; he, too, found that the Gottingen scheme is correct.

3. Mean gap length versus residual range

(a) The distribution of gap lengths

The distribution of gap lengths in tracks of ionising particles in nuclear emulsions has been investigated by C. O'Ceallaigh *et al.* [14, 15, 16] and P. H. Fowler and D. H. Perkins [17]. It is shown to be exponential for all measured values of the ionisation, i.e. the number of gaps with lengths between x and $x + dx$ is

$$f(x)\,dx = (1/\bar{G})\exp(-x/\bar{G})\,dx; \tag{1}$$

$f(x)$ is the frequency function of the distribution, $\bar{G}$ is the mean gap length.

This distribution is a special case of the Pearson type III distribution. The sample distribution of such a population is well known (M. G. Kendall: *The Advanced Theory of Statistics*, vol. 1, p. 244). The sample distribution for the mean value of a sample of size N is, if gaps $x < \varepsilon$ are excluded,

$$f(x)\,dx = \frac{1}{\Gamma(N)}\left\{\frac{N(x-\varepsilon)}{\bar{G}}\right\}^{N-1} e^{-\frac{N}{\bar{G}}(x-\varepsilon)}\,\frac{N}{\bar{G}}\,dx. \tag{2}$$

The mean value of this distribution is

$$\bar{G}_\varepsilon = \bar{G} + \varepsilon \tag{3}$$

and the standard deviation

$$\sigma = \bar{G}/\sqrt{N}. \tag{4}$$

The relations (3) and (4) have been experimentally verified by C. O'Ceallaigh [16].

The parameter $\bar{G}$, or $g = 1/\bar{G}$ as used by Fowler and Perkins, seems at present to be the best measure of ionisation in the track of a particle.

(b) The experimental determination of the mean gap length

Equation (3) makes it possible to measure $\bar{G}$ in a simple manner. If in a cell t with constant ionisation $\Delta L_\varepsilon =$ the total length of all gaps $\geq \varepsilon$ and $\Delta N_\varepsilon =$ the number of such gaps, then

$$\bar{G}_\varepsilon = \Delta L_\varepsilon / \Delta N_\varepsilon \tag{5}$$

is known and $\bar{G}$ can be determined.

Starting from (1) another procedure has been suggested [17]. If $N =$ the number of gaps $\geq \varepsilon$ per unit length and $B =$ all gaps > 0 per unit length or the blob density as usually defined, equation (1) yields

$$N = B\exp(-\varepsilon/\bar{G}). \tag{6}$$

Thus $\bar{G}$ can be determined by only counting N and B. As pointed out in the introduction this method is inapplicable at the ends of the tracks.

In this paper the first method is used. The choice of ε is influenced by several factors. The smaller ε the more information is obtained from the track, but the measurements are very tedious and the subjective errors larger, because one has to judge if a large number of small gaps are larger or smaller than ε. The larger ε the less information,

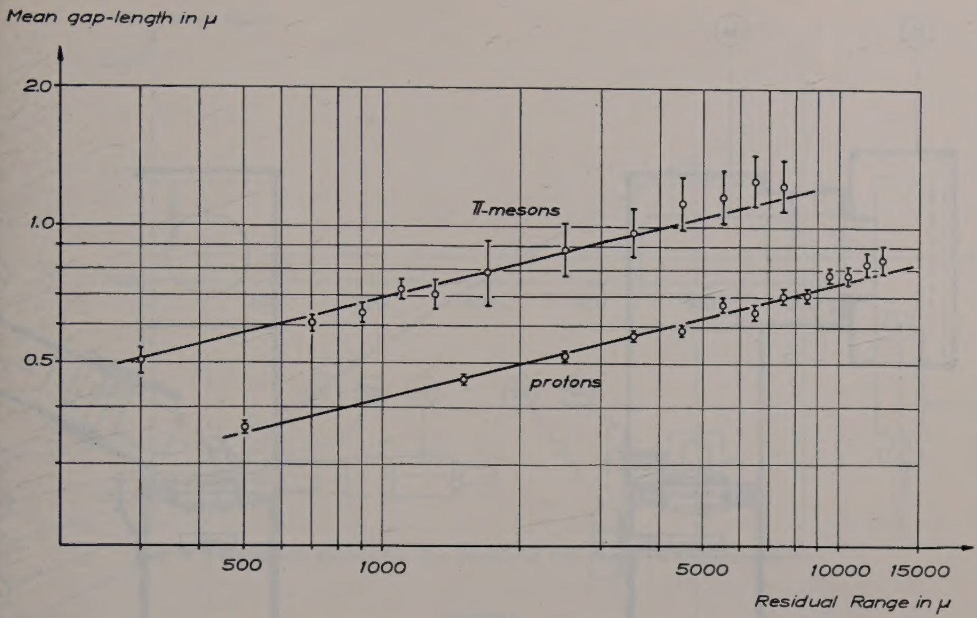


Fig. 6. The mean gap length as a function of residual range for protons and π-mesons.

but the speed is increased. ε has been chosen 0.292 μ in order to obtain a sufficient number of gaps in cells $t = 1000$ μ for the proton tracks.

The track is oriented along the y-axis of the microscope stage and moved forwards in the y-direction by the stage micrometer. ε is defined by two horizontal lines engraved on the glass plate of an eyepiece filar micrometer ([10] p. 54). The gaps are measured with the eyepiece micrometer. In order to facilitate the adjustment of the edges of the grains to the horizontal lines the micrometer of the y-axis is equipped with a worm gear wheel.

The mean gap length as a function of residual range is shown in Fig. 6.

The errors are standard errors calculated from equation (4). It is common practice to adopt the same slope for the linear regression line of the particle to be measured as for the reference line, in this case the proton line. Using this a weighted linear least square fit to the points gives,

$$\log \bar{G} = \beta_0 + \beta_1 \log R \tag{7}$$

for protons $\beta_0 = -1.1534 \quad \beta_1 = 0.2587$

for π-mesons $\beta_0 = -0.9340 \quad \beta_1 = 0.2587.$

These two lines are shown in the diagram.

For points with equal ionisation, i.e. the same $\bar{G}$, the ratio of the masses of the particles is the same as the ratio of the residual ranges. If (M, R) is the mass and residual range for an unknown particle and (M_0, R_0) the corresponding values for the reference particle, we have,

$$\frac{M}{M_0} = \frac{R}{R_0}. \tag{8}$$

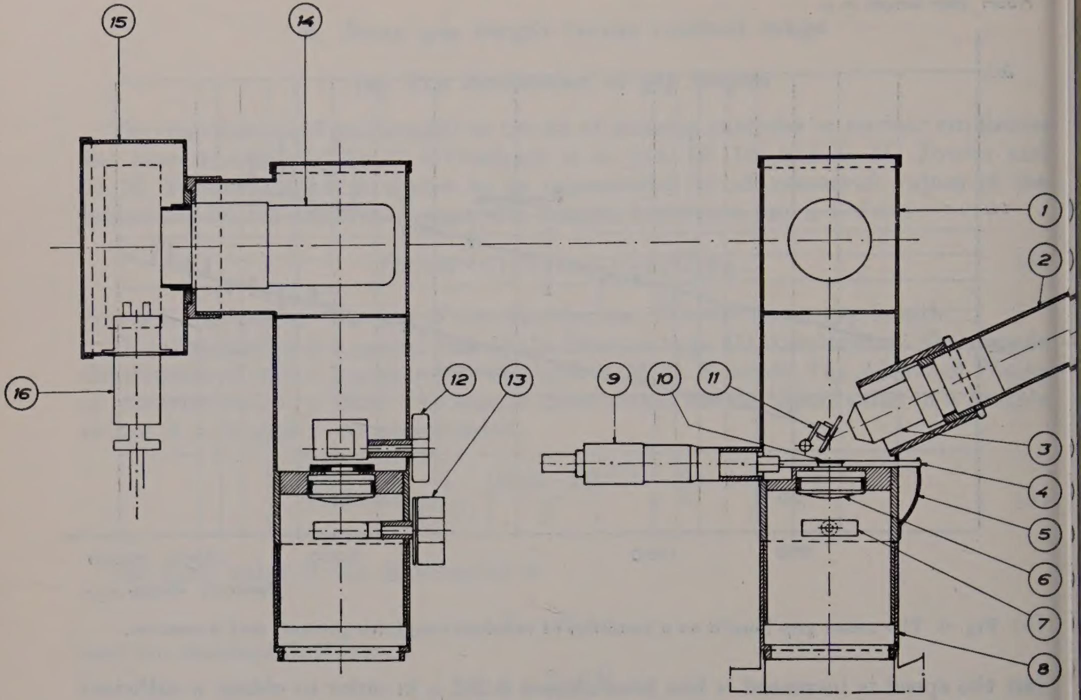


Fig. 7. The photo-electric apparatus.

1. The brass tube containing photo-multiplier
2. Eyelens of Cooke's comp. 8 × eyepiece
3. Objective, Cooke's 5 ×
4. Slide for the slit
5. Spring
6. Field lens of Cooke's comp. 8 × eyepiece
7. Movable glass plate
8. Brass tube screwed into the microscope
9. Micrometer screw for the slide
10. Slit
11. Movable holder with mirror
12. Spindle for turning the mirror
13. Spindle for turning the glass plate
14. Photomultiplier 1 P 21
15. Brass tube for the resistances of the photo-multiplier
16. Electric couplings to the multiplier

This yields $M_p/M_\pi = 7.0$ which may be considered satisfactory.

When comparing with the photometric measurements the gap coefficient $g = 1/\bar{G}$ in units mm^{-1} is used. This is normalised to that of minimum tracks yielding $g^* = g/g_0$. $g_0 = 291/\text{mm}$, measured in the same manner as the coefficient g.

4. Photometric measurement of ionisation

(a) The photo-electric apparatus

An apparatus for the photometric measurement of ionisation has been built. It is of the same type as that described by S. von Friesen and K. Kristiansson [18, 19, 20] but adapted for a Cooke microscope. The construction is shown in Fig. 7.

The photo-multiplier is directly coupled to a Speedomax G recorder. With an anode voltage of 750 V over the multiplier and a current of 150 μ Amps suitable readings are obtained on the recorder.

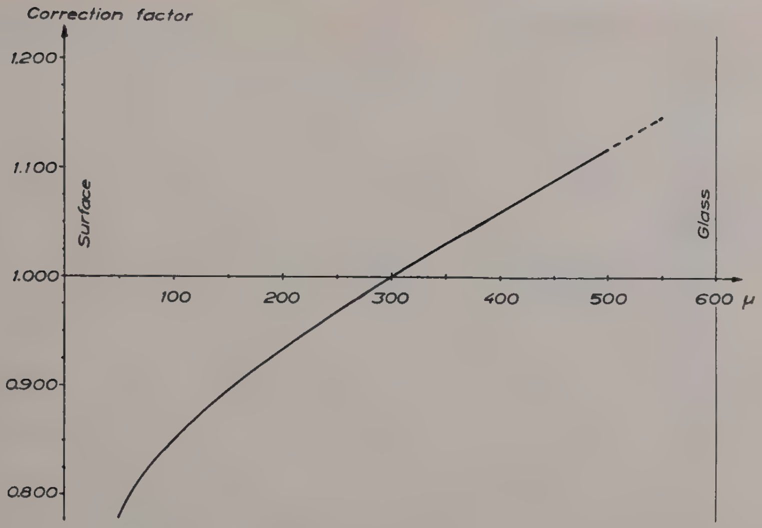


Fig. 8. Correction factor for the depth of the track in the emulsion.

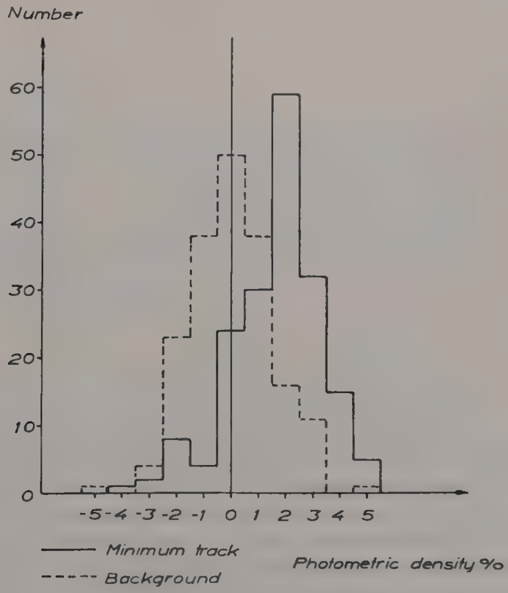


Fig. 9. The distributions of the photometric density for minimum tracks and background.

For the measurements a slit 0.3 × 3.0 mm corresponding to 2.9 × 29.0 μ in the object plane has been used. The turning of the plane parallel glass plate corresponds to a displacement of 5 μ of the slit on both sides of the track. The paper speed of the recorder is 12″/hour.

If S_b' is the scale deflection of the recorder with the slit on one side of the track, S_b'' with the slit on the other side and S_t with the slit on the track, the ratio

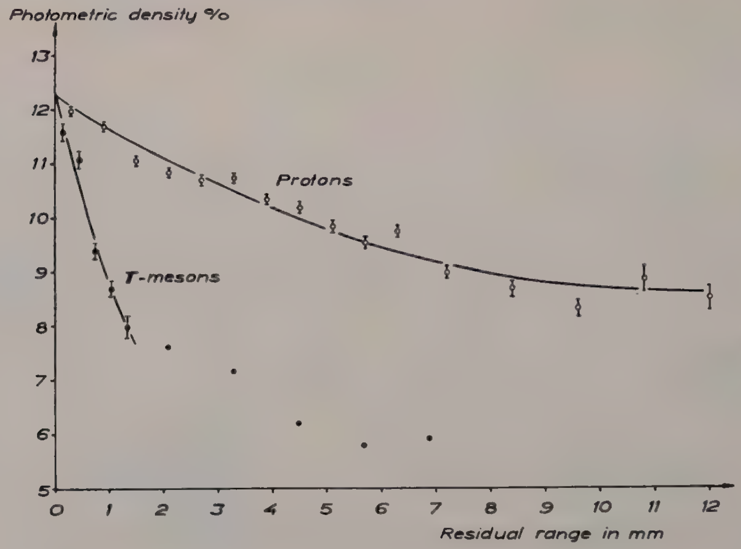


Fig. 10. Normalised photometric density as a function of residual range.

$$S_i = \frac{S_b' + S_b'' - 2\,S_t}{S_b' + S_b''}\,100\ \% \qquad (1)$$

is measured. The S_i are read in 30 μ intervals along the track and the mean value S is calculated in successive cells of 600 μ for protons and 300 μ for π-mesons. S is called the photometric density.

It is very important to correct for the depth of a track in the emulsion. The correction factor is the factor by which a reading should be multiplied in order to be normalised to the middle of the emulsion. The correction factor has been deduced from the first 600 μ of the proton tracks, which are situated at different depths. It is shown in Fig. 8.

It is in fair agreement with the factor deduced by B. Waldeskog [21], but not of the same general appearance as that by K. Kristiansson [22].

It is highly desirable to normalise the photometric density to that of minimum tracks in order to be able to compare the values obtained from different emulsions with different degrees of physical development and to compare the values obtained by means of other methods of measuring ionisation. This point has been investigated.

The distribution of 180 photometric densities along two minimum tracks from two negatron–positron pairs situated in the middle of the emulsion can be seen in Fig. 9. In the same figure distribution of 180 random readings from the same depth and in the vicinity of the tracks is shown.

The mean values are 1.40 % and 0.06 % respectively, the last value confirming that the mean value of random readings equals 0, as it should. The results show that it is possible to separate minimum tracks from the background with the slit dimensions used. If S_0 is the photometric density of the minimum tracks, the coefficient $S^* = S/S_0$ is an absolute measure of ionisation in the same sense as g^*, normalised grain density etc.

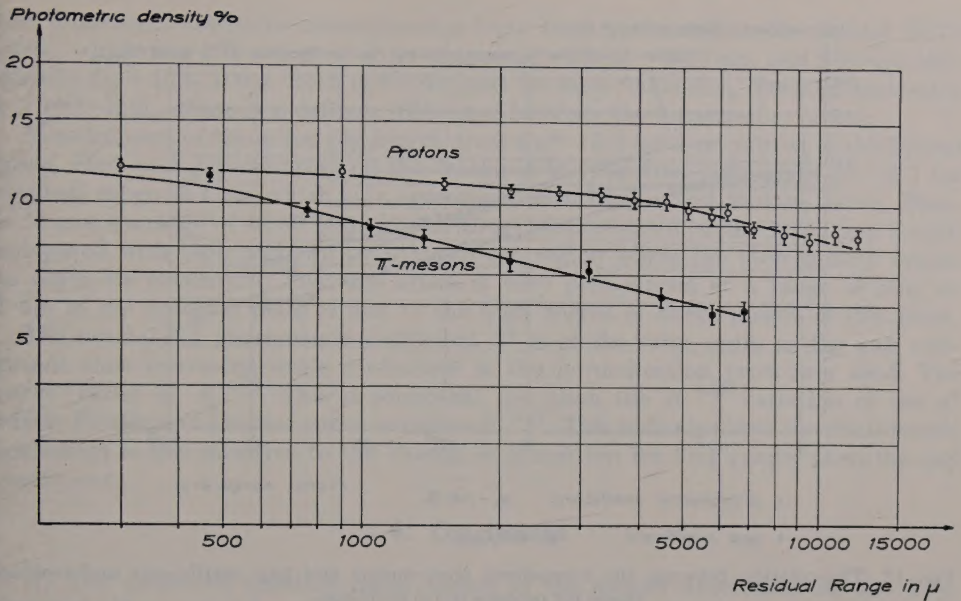


Fig. 11. The photometric density as a function of residual range in a log-log plot.

(b) The treatment of the measurements

The usual way to treat the measurements is to plot the photometric density S against residual range R. This is done in Fig. 10. The errors are standard errors derived from the spread of the experimental points.

A parabolic least square curve is fitted to the proton points giving a relation,

$$S = \beta_0 + \beta_1 R + \beta_2 R^2 \qquad (2)$$

with $\beta_0 = 12.32 \quad \beta_1 = -0.661 \quad \beta_2 = 0.0296$

S in per cent and R in mm.

The same calculation for the first five meson points yields $\beta_1 = -3.815$ $\beta_2 = 0.3772$. The curve is normalised to the same β_0 as the proton curve. By comparing the residual ranges of $S = 10.0$ in the middle of the measured points one obtains $M_p/M_\pi = 6.7$. This may be considered satisfactory and shows that the apparatus is working well. Only the first five points of the π-meson curve are used in order to obtain sufficient accuracy with a second degree expression. K. Kristiansson has shown [19] that a third degree expression is required for L-meson tracks about 1500 μ in length.

The above treatment is in fact limited to proton tracks less than 10000 μ and L-meson tracks less than 2000 μ in length, owing to the labour involved in calculating higher degree approximations.

In order to extend the applications of the photometric method and to bring it closer to other treatments of ionisation measurements the following procedure is suggested.

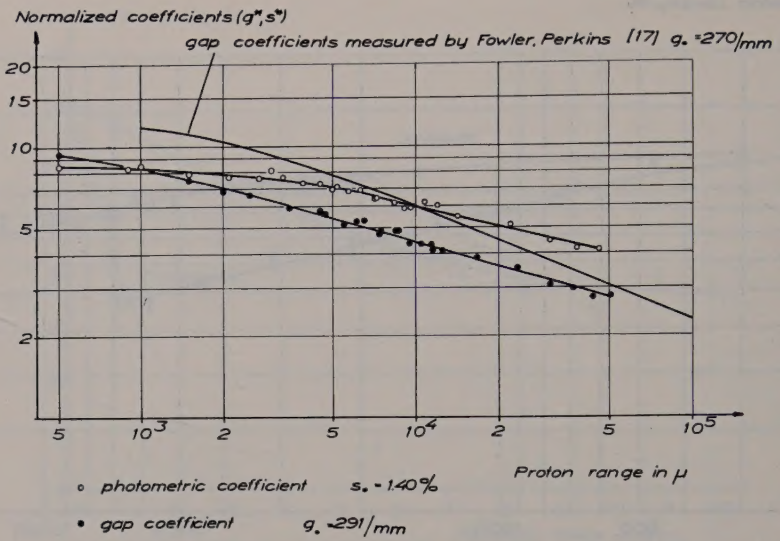


Fig. 12. The relation between the normalised photometric and gap coefficients and residual range for protons in G5 emulsion.

S is plotted versus R in a log-log diagram (no normalisation of the values from different particles). Fig. 11 shows that the points are very well approximated by a straight line, for π-mesons with a range $> 700\,\mu$ and for protons with a range $> 5000\,\mu$.

From a weighted least square fit one obtains a curve

$$\log S = \beta_0 + \beta_1 \log R \tag{3}$$

for protons $\beta_0 = 1.8243 \quad \beta_1 = -0.2241$

for π-mesons $\beta_0 = 1.6620 \quad \beta_1 = -0.2353.$

Thus the curves have very nearly the same slope. Adopting the slope of the π-line one can determine the M_p/M_π ratio at 7.4. This is a little too high but it must be kept in mind that these calculations are based on very few tracks. The possibility of treating the material in this way seems, however, to be established.

5. Comparison of the normalised ionisation measurements

The normalised coefficients g^* and S^* for protons are shown in relation to residual range in Fig. 12. The values from the measurements on the π-mesons are used by calculating the equivalent proton range assuming a mass of the π-meson of 273 m_e.

For comparison the gap coefficient measured by Fowler and Perkins [17] is shown in the diagram. They measured g^* by counting blobs and gaps exceeding 4.17 μ. They obtained identical g^* values for different emulsions for ranges longer than 1 cm. The difference between the g^* curves is a real effect, because the g^*—as measured by the method in this paper—does not differ between two different observers and the points are well outside the expected statistical deviation. In order to explain the difference, extensive measurements have been performed in the Bristol B27b plate, which has a low degree of development with $g_0 = 186/\text{mm}$ and plateau blob density $B_0 = 13.5/100\ \mu$. Both methods yield the same value of g_0. For high ionisation $g > 1000/\text{mm}$—there is a definite difference between the measured g values.

Measurement of the mean gap length gives a $g^* = 5.3$ for 1 cm proton in the Bristol plate. Blob and gap counting on the tracks in the Sardinia plate gives $g^* = 5.7$ for residual range of 1 cm which is in agreement with Fowler and Perkins curve. There is thus a shortage of small gaps in the direct measurement of the mean gap length compared with blob and gap counting in the region where the blob density begins to reach its maximum. That the effect is very pronounced at a range as long as 1 cm in the Sardinia plate is due to the high degree of development of this plate.

The normalised photometric coefficient S^* is of the same order as the gap coefficient thus increasing one's confidence in the normalisation procedure used. The curve varies as $R^{-0.23}$. This is somewhat less than the $R^{-0.26}$ variation of the g^* curve. Fowler and Perkins curve varies as $R^{-0.42}$. This indicates that the photometric coefficient is less sensitive to the change in ionisation for long ranges than the gap coefficient.

6. Conclusions

(1) The measured absolute mean values of the second differences for the Fay-Gottstein-Hain $p = 0.5\ \mu$ and $\pi = 0.5\ \mu$ scattering schemes are $p = 0.527 \pm 0.016\ \mu$ and $\pi = 0.543 \pm 0.026\ \mu$; cut-off without replacement being used. (2) The empirical conversion factors to be used when calculating the second differences from the third and fourth differences are 1.034 ± 0.027 and 1.095 ± 0.031 respectively. (3) The gap coefficient obtained by direct measurement of the mean gap length is smaller than the gap coefficient measured by blob and gap counting for $g > 1000/\text{mm}$. (4) The relation between the photometric density and residual range is linear in a log-log plot for proton ranges exceeding 5000 μ. (5) The normalised photometric coefficient is less sensitive to the change in ionisation than the gap coefficient for proton ranges exceeding 5000 μ.

ACKNOWLEDGEMENTS

I should like to express my sincere gratitude to Docent Gösta Ekspong for his encouragement and valuable advice during the course of this investigation. I also wish to thank Mrs. Ingeborg Backlund, who performed most of the tedious numerical calculations, and the Swedish Atomic Committee for financial support.

Institute of Physics, University of Uppsala, Sweden, October, 1955.

REFERENCES

1. W. T. Scott, Phys. Rev. *76*, 212, 1949.
2. S. Biswas, E. C. George, B. Peters, and M. S. Swamy, Suppl. Nuovo Cim. *12*, 369, 1954.
3. L. Vigneron, Journ. de Phys. et Rad. *14*, 145, 1953.
4. M. G. K. Menon and C. O'Ceallaigh, Phil. Mag. *44*, 1291, 1953.
5. S. Biswas, E. C. George, D. Lal, Yash Pal, and B. Peters; C. Dilworth, S. Goldsack, and L. Hirschberg: Proc. Bagnères Conf. 136, 1953.
6. T. Holtebekk, N. Isachsen, and S. O. Sorensen, Phil. Mag. *44*, 1037, 1953.
7. S. Biswas, E. C. George, and B. Peters, Proc. Ind. Acad. Sci. *38*, 418, 1953.
8. C. Dilworth, S. J. Goldsack, and L. Hirschberg, Nuovo Cim. *11*, 113, 1954.

9. H. Fay, K. Gottstein, and K. Hain: Suppl. Nuovo Cim. *11*, 234, 1954.
10. G. Ekspong, Ark. f. Fys. *9*, 49, 1954.
11. R. H. W. Johnston, Proc. Pisa Meeting (prov. ed.) 287, 1955.
12. M. Di Corato, D. Hirschberg, and B. Locatelli: Proc. Pisa Meeting (prov. ed.) 275, 1955.
13. S. Kaneko, Journ. Phys. Soc. Japan *16*, 325, 1955.
14. R. H. W. Johnston and C. O'Ceallaigh, Phil. Mag. *45*, 424, 1954.
15. M. G. K. Menon and C. O'Ceallaigh, Proc. Roy. Soc. *A 221*, 292, 1954.
16. C. O'Ceallaigh, CERN Report, B.S. 11, 1954.
17. P. H. Fowler and D. H. Perkins, Phil. Mag. *46*, 587, 1955.
18. S. von Friesen and K. Kristiansson, Ark. f. Fys. *4*, 505, 1952.
19. K. Kristiansson, Ark. f. Fys. *8*, 311, 1954.
20. S. von Friesen and L. Stigmark, Ark. f. Fys. *8*, 121, 1954.
21. B. Waldeskog, Ark. f. Fys. *7*, 475, 1954.
22. K. Kristiansson, Phil. Mag. *44*, 268, 1953.

Tryckt den 7 september 1956

Uppsala 1956. Almqvist & Wiksells Boktryckeri AB